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Technical Paper

The Quay Crane Scheduling Problem

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ABSTRACT

The recent growth in worldwide container terminals' traffic resulted in a crucial need for optimization models to manage the seaside operations and resources. Along with the recent increase in ship size and the container volume, the advancements in the field of Quay Crane Scheduling introduced the need for new modeling approaches. This is the motivation behind the current paper, which focuses on developing a novel yet simple formulation to address the Quay Crane Scheduling Problem (QCSP). The objective of the problem is to determine the sequence of discharge operations of a vessel that a set number of quay cranes will perform so that the completion time of the operations is minimized. The major contribution is attributed to the way that minimization is performed, which is by minimizing the differences between the container loads stacked over a number of bays and by maintaining a balanced load across the bays. Furthermore, important considerations are taken into account, such as the bidirectional movement of cranes and the ability to travel between bays even before completion of all container tasks. These realistic assumptions usually increase model complexity; however, in the current work this is offset by the novel simple objective. This paper presents a mixed-integer programming (MIP) formulation for the problem, which has been validated through multiple test runs with different parameters. Results demonstrate that the problem is solved extremely efficiently, especially for small problem sizes.

1. Introduction

The invention of standard-sized shipping containers in the late 18th century, also known as "containerization", has revolutionized the shipping industry. It has permitted a smoother handling of material goods and improved shipping operations that led to visualization of a smaller world, while it has also translated into novel magnitudes of income, allowing the world economy to experience unprecedented growth from this sector [13]. In order to sustain this growth, seaport terminals around the world are striving to keep up with the increase in the volume of containers' traffic: many of them are currently preparing to meet the challenge of handling ultra large container vessels capable of carrying 15,000 TEUs and beyond [18]. Baird [1] provided a brief insight into the continuous growth in ship sizes and overall traffic. However, this rapid increase in container traffic is not matched by an equal growth in seaport capacities, which poses a challenge for seaport terminals to overcome.

The increase in the number of container terminals (CT) worldwide comes hand in hand with the soaring competition factor. The customers are presented with a set of choices amongst competing service offers, where the terminal operators providing a higher caliber of services will deliver quality services that attract and retain customers by continuously meeting demand at all times. This made seaports strive to distinguish themselves from others by elevating the provided services to a platform that supports attracting a higher volume of ships, and a larger magnitude of shipments. Seaports are in a critical state where there exists a need to consider more efficient operations planning and management, thus a pending need arises for tailored solutions that address this challenge. Therefore, this area of research is receiving growing attention to help boost the performance of container terminals.

Seaports around the world are service providers that offer their facilities to ship and cargo owners. Their services can be divided mainly into seaside (quayside) services and landside (yard) services. These services include a set of loading/unloading operations as well as moving the cargo from the ship to the storage yard and vice versa. The seaside operations in particular involve the utilization of two critical resources, namely the quay space and the quay cranes (QC). Most of the literature addressing the seaside operations planning problems divides them into three independent problems which aim to optimize the utilization of these two resources: (1) the Berth Allocation Problem (BAP), (2) the Quay Crane Assignment Problem (QCAP), and (3) the Quay Crane Scheduling Problem (QCSP) Meisel [16]. The BAP has been more frequently addressed, such as in the work of Simrin and Diabat [21]

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and Simrin et al. [22], as well as that of Al Zaabi and Diabat [24]. The current paper tackles the problem of scheduling Quay Cranes, namely the Quay Crane Scheduling Problem (QCSP). Solving this problem to optimality can assist in shortening the time that vessels spend at the port, and in efficiently utilizing the available QCs, which are amongst the most expensive equipment in container terminals. The existing literature provides different ways of modeling the QCSP with ranging levels of detail; container groups, complete bays, and bay areas, as summarized by Bierwirth and Meisel [3]. The majority of these models have common assumptions such as non-preemption and unidirectional movement, which no longer reflects the practical scenario that accommodates the increasing ship size and QC capabilities in having it move at a higher speed. This serves as a motivation for the developed model in this paper.

The main contribution of the current paper lies in the novelty of the objective function, whose aim is to minimize the relative difference in remaining container workload between bays. To the best of our knowledge, this approach has not been adopted in past literature. However, the study performed in this paper concludes that the proposed objective leads to optimal crane schedules, in the sense that handling time is minimized and therefore lower costs are incurred for vessel operators, while the port resources are utilized efficiently. The important advantage of the current approach is the relaxation of the non-preemption assumption, which prevents the QC from moving between bays before finishing the assigned task completely. This accommodates a more realistic scenario that takes advantage of the flexibility, and hence produces a better solution. The QCSP is usually one of high complexity; hence, this paper proposes a simple yet accurate approach to generate the optimal solution through a simple formulation that is solved in a time efficient manner. The proposed formulation can be also extended to include the relaxed constraints (i.e. traveling time between bays) as will be described briefly in the formulation section.

In the following sections of this paper, a literature review on the QCSP is described in Section 2. Following that, Section 3 details the assumptions, the problem definition, and the novel formulation approach of the QCSP. In Section 4, a set of experiments was performed to evaluate the effectiveness of the proposed model. Finally, Section 5 concludes the study's findings and provides certain directions for future research.

2. Literature review

Providing container terminals with models and methods that lead to operational efficiency is undeniably essential to help seaports respond to the incrementing container streams through the universal supply chain system. Therefore, recent years witnessed an increasing number of research papers that aim to advance seaport operations. A useful classification of the existing models is presented by Bierwirth and Meisel [3]. In the current review, notable works will be described that cover the range of problem formulation types, mainly with respect to the objective function, and whose solution techniques generate satisfactory results for the QCSP.

The crane scheduling problem was initially addressed by Daganzo [6], who developed an MIP model for the loading of ships. His model assigned cranes to bays at specific time slots, in a way that ensured a balance for the total workload between cranes. The author proposes both exact and heuristic solution methods, while the objective of the model is the minimization of the total cost of delay incurred on the vessels, unlike what became prevalent thereafter, which is the minimization of the makespan required to complete tasks. Note that this primary work does not consider crane interference. The author solved small-sized instances of the model using a simple technique that he developed based on some optimality principles and some other common-sense observation. Later work of Peterkofsky and Daganzo [20] solved larger instances of the model using the branch-and-bound method. Zeng et al. [25] developed a mixed-integer programming model for quay crane dual-cycling scheduling. Their model considers the stowage plan of outbound containers and the operation sequence of quay cranes. They solved the model using a heuristic method, called bi-level genetic algorithm.

Lee et al. [12] proposed a formulation based on assigning bay areas to QCs, while assuming individual throughput rates for each crane. Their objective was the maximization of the total throughput and they propose several heuristics to solve it. In later work of Lim et. al [14] a formulation based on complete bays was proposed. They showed that there is always an optimal schedule among the unidirectional cranes when being assigned to complete bays. These formulations did not take detailed crane schedules into consideration nor did they use the minimization of the makespan as an objective. This can be found in the work of Kim and Park [11], in which the authors minimize the weighted sum of the makespan and the total completion time. In their work, however, clearance conditions are not enforced, i.e. constraints that guarantee a certain distance between adjacent QCs. This weakness was corrected by Moccia and Cordeau [19], who added these constraints, rendering the formulation more robust. Both formulations have since been referenced in numerous works. As far as solution methodologies are concerned, Kim and Park [11] suggested a branch-and-bound method to solve small-sized instances and a heuristic algorithm known as 'greedy randomized adaptive search procedure' (GRASP), in order to improve the performance of their branch-and-bound algorithm. Nonetheless, the authors did not discourse computational complexity that justifies the adoption of the heuristic algorithm developed. Succeeding Kim and Park's study, Liu et al. [15] considered the quay crane scheduling problem at container terminals where arriving vessels have different ready times, while Moccia et al. [19] solved instances which cannot be solved by Kim and Park's using a branch-and-cut algorithm.

One of the works that uses the MIP developed by Moccia and Cordeau [19] is that of Bierwirth and Meisel [2], in which the authors propose a heuristic solution procedure based on the branch-and-bound algorithm. The algorithm searches a subset of above average quality schedules and it exploits efficient criteria for branching and bounding, with respect to crane interference. The authors compared their approach to recent competing ones and they reported satisfactory results, especially for problem instances with a small number of cranes, with the significant advantage of reduced computational effort. According to the authors, the efficiency of the method can be attributed to the exclusive consideration of unidirectional schedules. Later work of Meisel [17] utilizes a different approach for Quay Crane Scheduling, where QCs are only available at certain time windows. The objective of the developed MIP is the minimization of total vessel handling time, determined by the latest completion time among all tasks. While the author solves this problem by searching the solution space of unidirectional schedules, he concludes that the optimal solution does not necessarily lie in the space of unidirectional schedules, but solutions are still of high quality.

The objective of the formulation of Liu et al. [15] is to minimize the maximum relative tardiness of vessel departures. In terms of model assumptions, the authors consider the aggregate workload of each bay, taken as the product of the number of containers to be handled in the bay and the average processing time per container, while vessels and the berth are partitioned into bays. This work does consider clearance constraints between adjacent cranes; productivity is assumed identical for all quay cranes and crane interference is ignored. The authors propose a heuristic decomposition approach to break down the problem into two smaller, linked models, namely the vessel-level model and the berth-level model. Both models are formulated as Mixed Integer Non Linear Programming (MINLP) problems, but the authors claim that they require less computational effort when compared to the MIP with many variables. Computational experiments show that the proposed approach is effective and efficient. Lee et al. [12] added handling priority for each ship bay to the MIP formulation for the QCSP and modified the objective function to minimize the sum of the weighted completion times of every ship bay. The work of Zhang and Kim [26] also introduces a different objective than that seen prevalent in the literature. Instead of minimizing the makespan, the aim of their MIP formulation is to minimize the total number of cycles of QC activities on one ship. They develop a heuristic algorithm, by decomposing the QC scheduling into an intra-stage optimization, which finds the optimal sequencing of all stacks in one bay, and an inter-stage optimization, which sequences all the bays. Numerical experiments reveal that the proposed approach found the optimal solution in most of the cases and greatly outperformed the scheduling methods used by port operators.

New set-covering formulations were introduced by Choo et al. [4], who propose a MIP for the crane sequencing problem and solve large-scale instances. They include clearance and yard congestion constraints, the latter being a relatively new addition to the QCSP considerations. In their work, the authors solve the MIP single-ship proposed model with the help of a heuristic approach that produces good results. The model is then reformulated as a generalized set covering problem and solved exactly by branch-and-price (B&P). For multiship sequencing, the yard congestion constraints are relaxed in the spirit of Lagrangian relaxation, so that the problem decomposes by vessel into smaller problems that can be solved by B&P at each iteration of the sub-gradient algorithm. The authors observe that computational results vary considerably but in all cases, the proposed Lagrangian relaxation technique is the recommended approach in terms of computational efficiency and solution quality for the multiship problems. Lagrangian relaxation was also implemented by Guan et al. [10], who obtained tighter lower bounds through this approach and reached a feasible solution within reasonable time.

Different work has been developed based on the scheduling formulation developed by Kim and Park [11] with emphasis on a new proposed solution technique. Chung and Choy [5] based their formulation on the one established by Kim and Park [11] and they suggested a modified Genetic Algorithm (GA) for solving the model. They proved that the proposed GA can achieve results very close to the best known solutions through comparing their results with a set of benchmarking data comprised of 43 instances. The best known solutions by other approaches were obtained for the same set by Tabu Search (TS) and branch-and-cut (B&C).

More recently, integrated models have evolved that consider the QCSP and the Quay Crane Assignment Problem (QCAP). The integrated problem simultaneously determines the assignment of quay cranes to vessels and the sequence of tasks to be processed by each quay crane. Fu et al. [9] proposed a formulation and heuristic solution approach to solve the integrated model, in which practical considerations are incorporated such as quay crane (QC) interference. A Lagrangian relaxation is proposed for the model in the later work of Fu and Diabat [8] and the authors report satisfactory results, thus recommending Lagrangian relaxation as a suitable approach for such problems. Theodorou and Diabat [23] transformed the integrated quay crane assignment and scheduling problem (QCASP) into a crane-to-bay assignment problem and introduced a Lagrangian relaxation algorithm to solve it. This was done based on their earlier work [7] that proposed a GA as a solution method. These integrated models emerged to take advantage of the interdependencies between the QCAP and the QCSP on the consequence of producing a basic schedule for the QCs.

It becomes evident that the scope of the crane scheduling problem is large, and researchers are still studying the different approaches both in terms of formulation and solution techniques, before reaching a consensus on the most accurate and time-efficient approach. However, there are certain realistic aspects that, after a certain point, are considered in all works. First of all, the aim of the QCSP is to determine the work schedule for cranes serving a vessel. QCs are heavy steel units that are mounted on a steel rail adjacent to each other and are used to load or unload containers from or to the vessel. Due to the single rail, the QCs cannot cross each other and need to maintain a safety distance. Typically, the vessel is divided into sections that are referred to as "bays" such that each bay can be serviced by one QC at most. Typically, QC's can travel from one bay to another once all the loading/unloading tasks on that bay are completed. The amount of time needed for a QC to move from one section to another is known as the traveling time. This paper devises a model that operates under the assumption that the traveling time is negligible, and that QCs can travel between the bays without completion of all tasks on that bay. The novelty of this paper is the objective of minimizing the load differences between bays. At the same time, it is assumed that cranes can travel between bays, even before the completion of all container tasks at the current bay. This assumption is usually relaxed, yet its consideration can significantly improve the optimal solution. Furthermore, unidirectional crane movement is not considered; this means that cranes can travel in both directions, rather than the common yet unrealistic approach of traveling only from left to right. It is expected that the proposed formulation will be significantly simpler than existing ones; however, without a compromise in accuracy, it is possible to generate an optimal solution for almost every problem instance. Furthermore, the advantage of low computational time outweighs several of the existing approaches, and it could set a new stream of problem formulations, to be further implemented and tested by researchers.

3. Problem description and formulation

3.1. Problem description

This section explores the problem's characteristics and the modeling assumptions that were adopted in this paper. Firstly, it is typical in seaside operations to partition the vessel into bay areas, each carrying a certain number of containers and indexed sequentially along the quay, according to their position from left to right. At any point in time, a QC can be assigned to at most one bay, which implies that each QC can perform the unloading and/or loading of containers only at the assigned bay. A single QC can handle at most one task at an instant in time; in other words a single QC can only perform the loading or unloading of a single container.

As far as QC movement is concerned, the most important physical consideration is preventing the crossing of cranes, given the fact that they are mounted on a single rail. This implies that, assuming cranes are also indexed sequentially along the quay from left to right, lower indexed cranes cannot be positioned to the right of higher indexed cranes, that is maintaining the sequence of the cranes' rank, as will be seen in a subsequent example. Furthermore, unlike the majority of models developed in the literature, the current model allows for QCs to travel between bays, even before the container tasks at that bay are completed. In fact, this is readily implemented in practice because this extra degree of freedom allows for better solutions. This leads to a model that better reflects real-life circumstances, as long as the non-crossing constraints are maintained at all times. Given the fact that the time required for cranes to travel between bays is negligible, compared to the handling times, it is not taken into consideration. However, this is easily incorporated as an extension to the model as illustrated in Section 3.

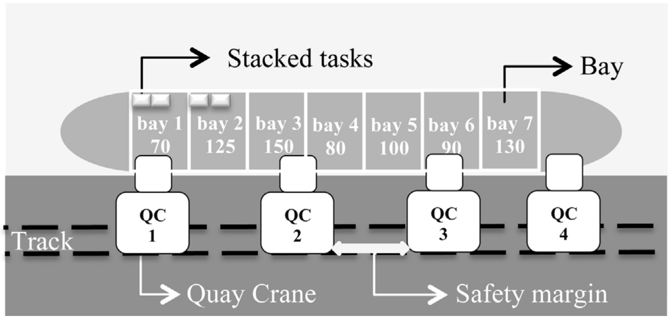


Fig. 1. Graphical illustration of the QCSP with 1 vessel, 7 bays and 4 QCs.

Another restriction that is adopted by several authors is the unidirectional crane movement, which enforces that all cranes travel in a single direction, without the possibility of moving backwards. In the present paper, this assumption is relaxed, as it is considered an unnecessary limitation to the formulation and it would not allow for fully taking advantage of the assumption that cranes can travel between bays, even before the completion of all container handling at the currently assigned bay. Therefore, the current model is bidirectional, giving QCs the freedom to travel both right and left, as long as they do not cross each other, and maintain the required safety margin, which is also practically possible. Finally, identical service rates are assumed for all cranes, but it can also be easily modified to account for variability in service rates. Fig. 1 provides a graphical illustration of the problem, depicting the sequential indexing along the quay for both bays and cranes, the rail upon which cranes are mounted, the safety margins between adjacent cranes, as well as the number of stacked containers on the bay. When dividing the vessels into bay areas, the safety margin between two adjacent QCs can be implicitly taken into consideration; hence the safety margin has not been incorporated into this model.

It can be observed from the literature review that typically the objective of the QCSP is to minimize the makespan of the QC schedule, which is measured as the latest completion time among vessels. However, it was noticed that this objective could be equally served by ensuring sufficient balance of the workload distribution among the cranes. Therefore, changing the objective function from the minimization of the makespan to the minimization of the relative differences in workload amongst all bays, will also indirectly minimize the makespan through ensuring a balanced container workload.

3.2. Problem illustration

In this section, a simple example was constructed to demonstrate that our novel approach yields an optimal solution. A single vessel is considered which is divided into three bays, each carrying 100, 150, and 125 containers respectively, as depicted in Fig. 2. The vessel in this example has been assigned two QCs, as a result of the Quay Crane Assignment Problem (QCAP), and now the aim of the QCSP is to generate the optimal crane schedule to unload all containers. Without loss of generality, we assume a container handling rate for QCs at 25 containers per hour and the objective is to best utilize the resources in order for all tasks to be completed within the least possible amount of time.

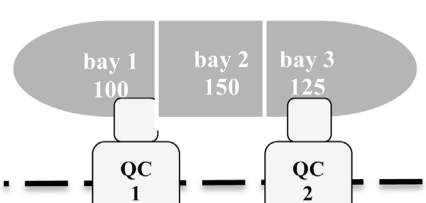
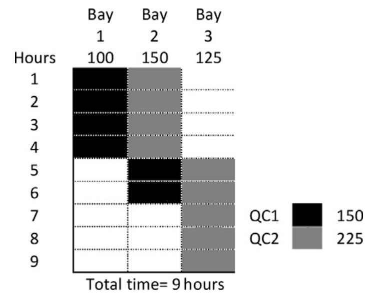


Fig. 2. Simple example with 1 vessel, 3 bays and 2 QCs.

Table 1
Proposed solution A.



Based on the assumptions of the new formulation, a set of possible solutions was constructed to reach an optimal solution for this simple example.

The first possible solution is the initial assignment of QC1 at bay 1 and QC2 at bay 2. After 4 h, all tasks on bay 1 are completed and then QC1 moves to bay 2, while QC2 moves to bay 3, without allowing the two cranes to cross each other. After allowing 3 h to pass, QC2 completes all its tasks which results in a total handling time of 9 h, as shown in Table 1.

Another possible solution is to start by placing QC1 at bay 2 and QC2 at bay 3 as illustrated in Table 2. In this case QC1 begins at bay 2, and 4 h later it moves backwards to bay 1, while QC2 stays in bay 3. An hour later, bay 3 is completed, while bay 1 has 3 h left and bay 2 has 2 h of work left. Therefore, QC2 moves to bay 2 while the work continues as is in bay 1. Following that, QC1 completes all tasks at bay 1 within 4 h and this results in a total time of 8 h, which is lower than the time required in the first case.

It can be noticed that solution B resulted in shorter time due to the better utilization of the resources. For simplicity, this problem can be viewed as the best allocation of resources that serve the bays while taking into consideration the practical constraints. For this specific example, following this logic can lead to the best possible completion time if the two QCs were fully utilized all the time. This would lead to a total number of containers to be processed equal to 375. Assuming 2 QCs, we obtain approximately 188 containers per QC.

In the third scenario, each QC would handle about 188 containers at optimality to keep them working all the time and to minimize the time in which they will be idle. Therefore, the QCs can be assigned to bays in a way that ensures that one QC will handle 188 containers and the other one will handle 187 containers. This can be achieved in 7 h and 31.2 min. In practice, this will require switching the QCs' location more often, but since the traveling time is overlooked for the time being, then the switching frequency does not play an important role. This solution is illustrated in Table 3.

Table 2
Proposed solution B.

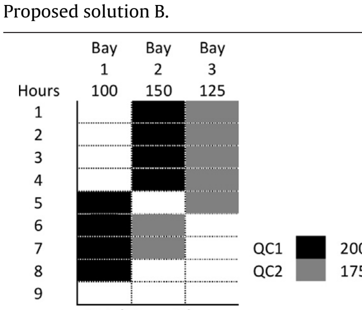
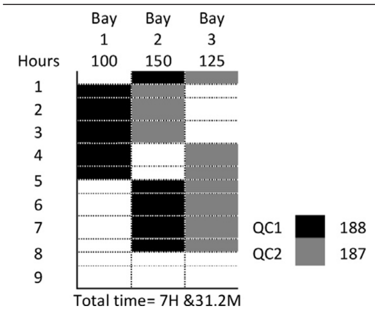


Table 3
Proposed solution C.



Exploring this simple version of the problem and the gradual evolution of its optimal solution has motivated us to develop a new model where we focus on the idea of utilizing the available resources at best case scenarios (equally distributed load) to produce the optimal work schedule. The model formulation is provided in the following section.

3.3. Optimization model

The notation that was used in the proposed formulation of the problem is shown below:

Sets:

J	$\triangleq$	set of bays, indexed by j or j'
I	$\triangleq$	set of quay cranes assigned to the vessel, indexed by i or i' where i or i' is the QC number in ascending order from left to right
T	$\triangleq$	set of time segments, indexed by t or τ

Parameters:

ω_j	$\triangleq$	workload in containers at bay j to be handled
μ	$\triangleq$	the identical rate of operation for QC
M	$\triangleq$	BigM, sufficiently large number

Decision variables:

$D^t_{jj'} \triangleq$ the difference between the workload existing in bay j and bay j' at time t

$x^t_{ij} \triangleq \begin{cases} 1 & \text{if quay crane } i \text{ is assigned to bay } j \text{ at time } t \\ 0 & \text{otherwise} \end{cases}$

$w^t_j \triangleq$ the unhandled workload at time t in bay j

The following is the nonlinear mixed integer formulation of this model.

$$\min \sum_j \sum_{j'} \sum_t |w^t_j - w^t_{j'}| + \sum_i \sum_j \sum_t t x^t_{ij} \tag{1}$$

Subject to:

$$\sum_j x^t_{ij} \le 1 \quad \forall i \in I;\ \forall t \in T \tag{2}$$

$$\sum_i x^t_{ij} \le 1 \quad \forall j \in J;\ \forall t \in T \tag{3}$$

$$\mu \sum_t \sum_i x^t_{ij} \ge \omega_j \quad \forall j \in J \tag{4}$$

$$w^t_j \ge \omega_j - \mu \sum_{\tau<t} \sum_i x^\tau_{ij} \quad \forall j \in J;\ \forall t \in T \tag{5}$$

$$\sum_{i'>i} \sum_{j'<j} x^t_{i'j'} \le M(1 - x^t_{ij}) \quad \forall i \in I;\ \forall j \in J;\ \forall t \in T \tag{6}$$

$$x^t_{ij} \in \{0, 1\} \quad \forall i \in I;\ \forall j \in J;\ \forall t \in T \tag{7}$$

$$w^t_j \ge 0, \quad \forall j \in J;\ \forall t \in T \tag{8}$$

In the formulation presented above, the objective function (1) minimizes the absolute value of the sum of the differences in workload over time between all bays. In addition, it contains a second term that prevents the allocation of cranes in higher order time segments. This nonlinear function can be linearized, as will be explained later with the help of equations (9), (10), (11) and (12). Constraints (2) ensure that each crane can be assigned to at most one bay in any given time segment, while constraints (3) guarantee that each bay is handled by at most one QC in any given time segment. Constraints (4) make sure that cranes will adequately be assigned to complete all tasks at every bay. Constraints (5) define the remaining workload at each bay at any specific time segment as the initial workload removing the processed work at the earlier time segments. Constraints (6) are the non-interference constraints, which take into account the ascending order of bays and quay cranes to prevent higher order cranes from being positioned to the left of lower order cranes, hence maintaining the rank. Constraints (7) and (8) restrict the domains of the decision variables.

The following is the linear formulation of the mixed integer program with an additional term in the objective function and three additional constraints.

$$\min \sum_j \sum_{j'} \sum_t D^t_{jj'} + \sum_i \sum_j \sum_t t x^t_{ij} \tag{9}$$

Subject to:(2)–(8)

$$D^t_{jj'} \ge w^t_j - w^t_{j'} \quad \forall j, j' \in J;\ \forall t \in T \tag{10}$$

$$D^t_{jj'} \ge w^t_{j'} - w^t_j \quad \forall j, j' \in J;\ \forall t \in T \tag{11}$$

$$D^t_{jj'} \quad \forall j, j' \in J;\ \forall t \in T \tag{12}$$

In the linear formulation above, the objective function described by Eq. (9) is to minimize the differences between the workload between all bays at all times, thus maintaining a balanced workload as much as possible, similar to Eq. (1). However, it introduces a new term to allow for the linearization of the function, by additionally defining constraint sets (10) and (11), in addition to the second term that prevents the allocation of cranes in higher order time segments. Finally, constraints (12) restrict the domains of the added decision variables. The model above has chosen to ignore the traveling time since it is relatively small comparing to the load that is distributed over the bay, given the advancement of the QC equipment used. However, the traveling time can be easily included by defining a new variable and adding a constraint that accounts for the traveling time, and then removing the second term of the current objective function as detailed below.

Additional parameters:

$\lambda^i_{jj'} \triangleq$ the traveling time of quay crane i between bay j and bay j'

Additional decision variables:

$y^t_{ijj'} \triangleq \begin{cases} 1 & \text{if quay crane } i \text{ is moved from bay } j \text{ to bay } j' \text{ at the end of time } t \\ 0 & \text{otherwise} \end{cases}$

Modified objective function:

$$\min \sum_j \sum_{j'} \sum_t D^t_{jj'} + \sum_i \sum_j \sum_{j'} \sum_t \lambda^i_{jj'} y^t_{ijj'} \tag{13}$$

Table 4
More QCs, with lower operation rates.

Experiment number	Base	1	2	3	4	5
Number of QCs	4	5	5	6	6	6
Rate of Operation (container/hr)	30	15	20	15	20	30
Required time to finish all tasks (hr)	7	10	8	9	7	5

Table 5
Fewer QCs, with higher operational rates.

Experiment number	Base	1	2	3	4
Number of QCs	4	3	3	2	2
Rate of Operation (container/hr)	30	35	60	35	60
Required time to finish all tasks (hr)	7	8	6	12	9

Additional constraints:

$$y_{ijj'}^{t} \geq x_{ij}^{t} + x_{ij'}^{t+1} - 1 \quad \forall j, j' \in J; \forall t \in T \backslash t \leq |T| - 1 \tag{14}$$

$$y_{ijj'}^{t} \in \{0, 1\} \quad \forall j, j' \in J; \forall t \in T; \forall i \in I \tag{15}$$

The previously presented constraint has been included to demonstrate the flexibility that this approach offers for adding travel time to the model. Additionally, constraints (13)–(15) have not been included in the results and analysis.

4. Computational analysis

In this section, the computational analysis that was performed is presented to draw conclusions regarding the problem characteristics, as well as to evaluate the effectiveness of the proposed approach. This preliminary computational analysis consists of examining the effect of several factors, such as the number of available QCs, their rate of operation, the distribution of the total workload over the bays, and the number of bays on the time required to complete all tasks. Initially, the base case experimental setting is presented, where it is used as a benchmark for the other results. In the base case setting, there are 4 QCs, each with an operational rate of 30 containers per hour. In this theoretical case, the vessel consists of 7 bays with the container workload distribution shown in Fig. 1.

In the following sections, the effect of different factors is studied, such as the number of QCs, their rate of operation, the distribution of workload over the bays, the number of bays, etc. on the time required to finish all tasks.

4.1. Effect of varying the number of QCs and their associated rates of operation

The first factor that is studied is the effect of a varying number of QCs and different rates of operation, for the various numbers of QCs. It is obvious that an increase in the number of QCs would lead to a decrease in the time required to handle all containers. Similarly, an increase in the operational rate would lead to a decrease in the handling time. Therefore, both effects were studied for two cases: (a) increasing the number of QCs and reducing their operational rate, and (b) decreasing the number of QCs but increasing their operational rate.

Table 4 refers to case (a) for which four cases were examined and compared to the base case, which is the highlighted column of the table. For the two first experimental settings, the number of QCs was increased by 1, and the results were generated for an operational rate of 15 and 20 containers per hour, respectively. For the next two settings, the performed change was an additional increase in the number of QCs by 1, and the operational rates of the two first settings were maintained.

- Case (a): more QCs, with lower operation rates

The results essentially demonstrate that adding more QCs that are slower in terms of operational performance does not improve the handling time of the vessel; rather, it results in an increase in the time required. In the cases with the lowest operational efficiency, the increase is significant, up to almost 60%. Of course, this could have to do with the fact that four containers working at an operational rate of 30 containers per hour can ultimately lead to a handling rate of 120 containers per hour, while five cranes operating at 20 containers per hour leads to a peak rate of 100 containers per hour. However, even in the case where there are 6 QCs at 20 containers per hour, which implies the same maximum rate as the base case, the handling time increases.

- Case (b): fewer QCs, with higher operational rates

Table 5 refers to case (b) for the which the number of QCs was decreased in comparison to the base case by 1 for the first two experimental settings and by 2 for the next two settings. The operational rates for each set are 35 and 60 containers per hour. In this case, it was noticed that using fewer more efficient cranes can lead to an improvement in the time, such as in the second experimental setting; however, it can also cause significant delay, if the number of QCs is reduced below a certain threshold, such as in setting 3 and 4.

4.2. Effect of varying the distribution of the same total workload over bays

In this part of the analysis, the effect of redistributing the total container workload among the bays was examined. As a measure of the distribution, the standard deviation was employed, which was obtained by the root of the sum of squared differences in container workload between all bays.

In Fig. 3, the red triangle refers to the base case, which corresponds to a standard deviation of 29.25. The extreme point with a standard deviation of 281 refers to the case where the whole container workload is distributed to a single bay. It is easy to observe that the higher the standard deviation, which translates into greater differences in container workload between bays, the higher the time required to complete the handling of containers. In fact, a substantial rise in the handling time was observed, which implies that an uneven distribution of containers can cause great delays.

4.3. Effect of reordering workload over the same bays

At this point, the effect of reordering the container load over the same bays was studied in three ways. Firstly, the container workload was placed in an ascending order, meaning that higher indexed bays carry more containers than lower indexed bays. In the second

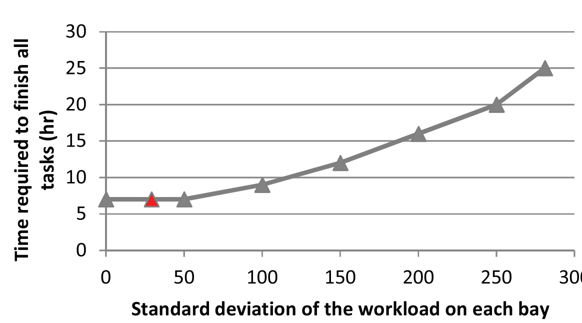


Fig. 3. Effect of varying distribution of the same total workload over bays.

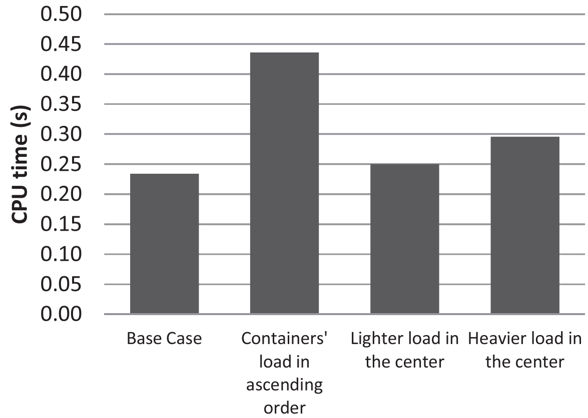


Fig. 4. Effect of reordering workload over same bays.

Table 6
Effect of increasing problem size on CPU time.

	QCs #	Bay #	Equations #	Variables #	Discrete Variables #	CPU
RUN 1	4	7	3258	1926	700	0.234
RUN 2	7	10	6936	4251	1750	0.921
RUN 3	9	10	7486	4751	2250	2
RUN 4	4	14	11,315	6301	1400	4
RUN 5	7	14	12,440	7351	2450	5
RUN 6	9	14	13,190	8051	3150	10
RUN 7	12	14	14,315	9101	4200	21
RUN 8	10	18	20,969	12,601	4500	83

setting, the highest number of containers was placed in the central bay, while in the third setting the minimum number of containers was placed in the central bay. Fig. 4 demonstrates the results in terms of handling time as well as CPU time.

An interesting result was found where the handling time for all three experimental settings was found to be identical to the handling time of the base case, which is equal to 7 h. This means that the ordering of the container workload does not play a significant role in the time required to perform all tasks. However, the ordering does impact the CPU time, i.e. the time required for the model to reach the optimal solution. This may be negligible for small-sized problems such as the current one, but in larger instances it could make a great impact.

4.4. Effect of increasing problem size on CPU time

In this final analysis, the effect of greater problem sizes on the CPU time was studied. Table 6 summarizes the important problem characteristics, such as the number of QCs and the number of bays, as well as important model characteristics, such as the number of equations, variables and discrete variables, for the different runs that were performed. From Fig. 5, as was expected, increasing the problem size leads to an increase in the required CPU time. However, what is interesting to observe is the fact that this increase is exponential. This constitutes our method very efficient for small problem instances, but not as efficient as for greater sizes.

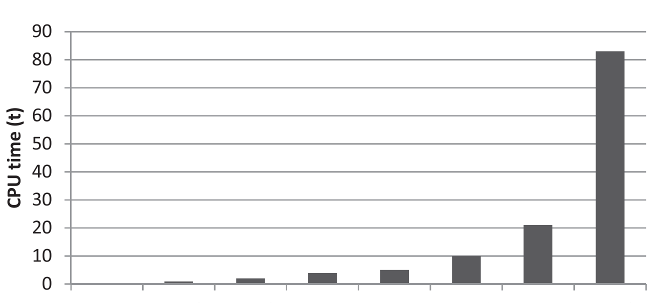


Fig. 5. Effect of increasing problem size on CPU time.

5. Conclusions

The importance of this work lies in the novelty of the proposed approach. Despite the fact that numerous formulations have been presented in the literature, this approach has yet not been considered; devising an objective function that is based on minimizing the relative difference in container workload between bays for the Quay Crane Scheduling Problem (QCSP). Therefore, the present paper addresses this gap, by introducing a simple yet accurate formulation that takes into consideration the real-life circumstances pertaining to this problem. These include bidirectional crane schedules and the ability of cranes to move between bays, even before completion of handling the current bay.

After presenting the proposed formulation, a computational analysis is conducted in order to evaluate different scenarios which can lead to useful conclusions regarding technical characteristics of the problem. Specifically, the first analysis comprises of increasing or decreasing the number of cranes with a respective decrease or increase in their operational efficiency. This analysis leads to results which demonstrate that a lower number of cranes with higher efficiency is the preferred option. Furthermore, changing the distribution of the container workload among the bays has an impact on the total handling time required to serve the vessel. It is shown that the greater the standard deviation of the container workload difference between bays, the greater the handling time required. Another interesting observation that arises from the present work has to do with the reordering of containers on the bays. Results clearly demonstrate that, while reordering does not have an impact on the handling time, it does have an impact on the CPU time. For small problem instances this may not be significant, but it does become so when applied to real-life problem sizes. In addition, the effect of increasing the number of bays, while maintaining the same workload and the same number of cranes was examined. The conclusion was that a larger number of bays did not lead to a lower handling time, which may seem counter-intuitive, given the fact that more bays allow for more movement for the QCs. Finally, increasing the problem size leads to an exponential increase in CPU time, which constitutes this model ideal for small and medium sized problems.

As far as future research is concerned, there is great potential for expansion of the current model. Even though the formulation is simple, there is a complexity inherent to the scheduling problem, which arises from the non-crossing constraints, among others. Thus, a heuristic approach can be developed based on this model in order to produce results even more efficiently for large problems, and to underscore the model's superiority in terms of simplicity and computational efficiency. Furthermore, there are several assumptions that can be easily included, such as a non-constant productivity rate for cranes and the consideration of the travel times required for cranes to travel between bays. Finally, conducting tests on real-life instances that have been used by other works and benchmarking them would be extremely beneficial in terms of comparatively evaluating the model and ultimately judging its appropriateness for use by container terminal operators.

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